

INDIVIDUAL AND COLLECTIVE MATHEMATICAL AGENCY IN URBAN CLASSROOMS

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What generalization about addition is illustrated by these pairs of equations?

$$8 + 5 = 13$$

$$8 + 6 = 14$$

$$23 + 15 = 38$$

$$23 + 16 = 39$$

$$8 + 5 = 13$$

$$9 + 5 = 14$$

$$23 + 15 = 38$$

$$24 + 15 = 39$$

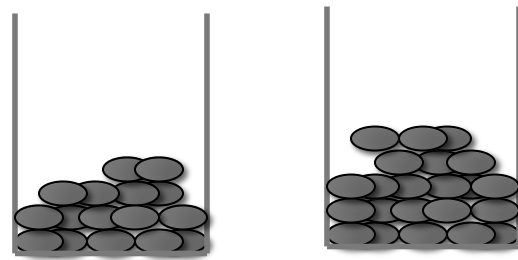
Students' written observations

- In the first column, if the number goes up, the answer goes up.
- The first number goes up by 1 and the second number stays the same, so the last number goes up by 1.
- One number grows by 1 and the sum grows by 1.

Class conjecture

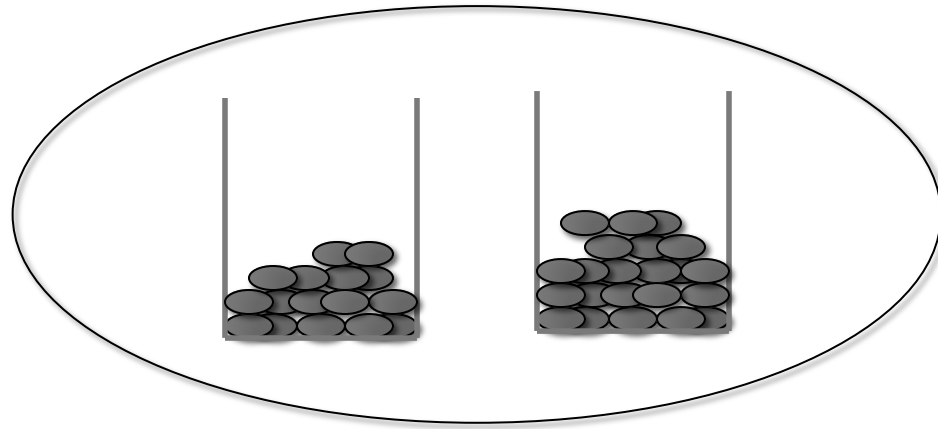
Given an addition expression, when one addend increases by 1 and the other stays the same, the sum increases by 1.

Representation-based argument



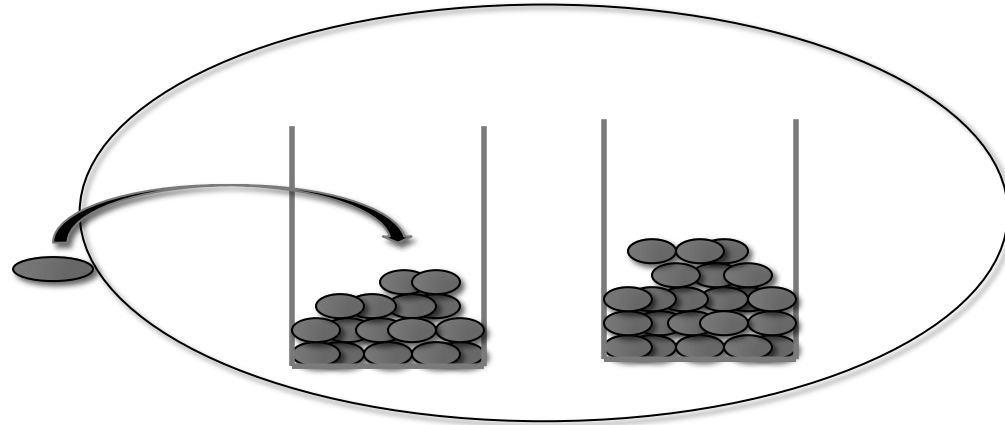
Addends are represented by two collections. It doesn't matter how many objects are in each of the jars.

Representation-based argument



The sum is represented by the joining of the two quantities.

Representation-based argument

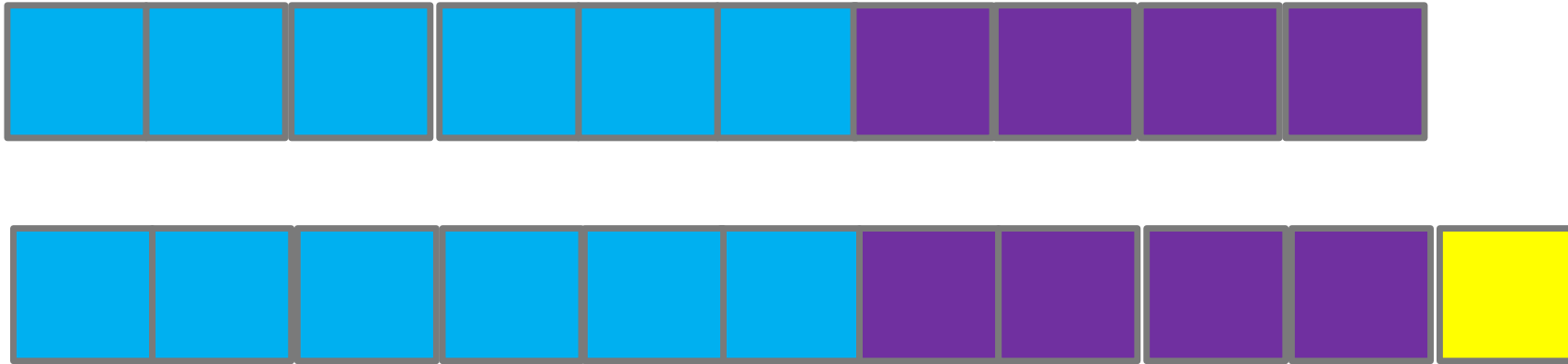


When 1 is added to one of the quantities (addends), the sum necessarily also increases by 1.

Representation-based argument



Representation-based argument



1. State a generalization about multiplication that is illustrated by these pairs of equations

$$7 \times 3 = 21$$

$$8 \times 3 = 24$$

$$7 \times 3 = 21$$

$$7 \times 4 = 28$$

“When one factor of a multiplication expression increases by 1, ...”

2. Create a representation to show why your generalization must be true.

Mathematical Agency

Mathematical agency—also referred to as “critical agency” (Turner, 2003) or “conceptual agency” (Gresalfi et al., 2009)—refers to students seeing themselves as active and powerful thinkers who engage in reasoning and construct new understandings for themselves.

Mathematical Agency and Underserved Students

Mathematical agency, closely related to mathematical identity, cannot be taken for granted, especially for students who have been historically excluded from opportunities to progress toward higher-level mathematics.

“Creating opportunities for students—particularly those who have traditionally had less access to powerful mathematics and mathematical practices—to engage in productive forms of agency should be a goal for all teachers [Aguirre et al., 2013, p. 17].”

Collective Mathematical Agency

“[T]eachers and their students act together to solve problems, working from the shared belief that viable strategies can be developed and solutions can be found. Different students can contribute different elements to this collective agency ... [including] productive reasoning strategies ... explanations of particular mathematical concepts or ... questions that help to clarify problems and concepts [p. 17].”

Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Are there ways in which you and your colleagues were engaged in individual and collective math agency in the activity we just did?

Classroom video: Individual and collective agency

- Where do you see individual mathematical agency?
- Where do you see collective mathematical agency?
- How do these examples of agency move the mathematics forward?
- What do you see the teacher doing that is supporting individual and collective agency on the part of the students?

Grade 3 Video: Adding 1 to a factor

$$7 \times 3 = 21$$

$$8 \times 3 = 24$$

$$7 \times 3 = 21$$

$$7 \times 4 = 28$$

Emmanuel Fairley-Pittman's 3rd grade

- Jaslanie: When I add 1 to the first factor, the product gets bigger because we add 1 from the 7 to the 8.
- Kevin: When I add 1 to the factor, the product increases by 3.
- Kedly: When I add 1 to the first factor, the product increases by how much we're multiplying by.

Emmanuel Fairley-Pittman's 3rd grade

Individual and collective agency

- Where do you see individual mathematical agency?
- Where do you see collective mathematical agency?
- How do these examples of agency move the mathematics forward?
- What do you see the teacher doing that is supporting individual and collective agency on the part of the students?

Michelle Sirois' 4th grade: class conjecture about multiplication

When I add a 1 to the first or second factor, the
product goes up by the factor that does not change.

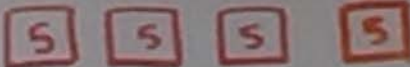
Changing 3 x 5 to 4 x 5: four representations


When I add a ~~part~~¹ to the first or second factor, the product goes up by the factor that does not change.


$3 \times 5 = 15$
 $4 \times 5 = 20$

$3 \times 5 = 15$
 $3 \times 6 = 18$

Jeff bought 3 bags of apples with 5 in each. Jeff's mom bought 1 more bag with 5 in it.







Core Questions about each representation

- Where do you see the original factors?
- Where do you see the original product?
- How does the representation show multiplication?
- Where do you see one factor increase by 1?
- Where do you see the increase in the product?
- How does the representation show the conjecture?

$$3 \times 5 = 15$$

$$4 \times 5 = 20$$

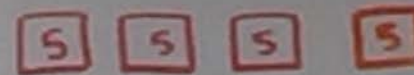
JEFF BOUGHT 3
BAGS OF APPLES
WITH 5 IN EACH.
JEFF'S MOM
BOUGHT 1 MORE
BAG WITH 5 IN IT.

When I add a ~~part~~¹ to the first or second factor, the product goes up by the factor that does not change.

$$3 \times 5 = 15$$
$$4 \times 5 = 20$$

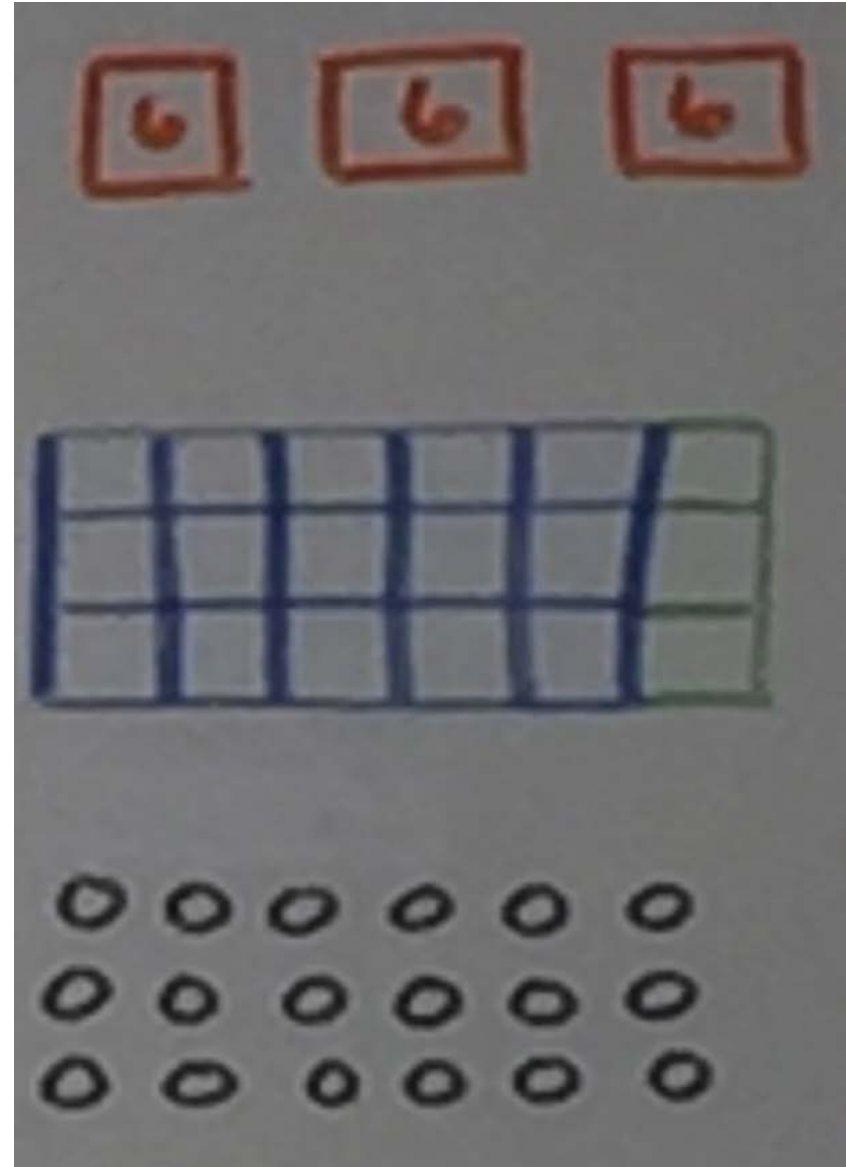
$$3 \times 5 = 15$$
$$3 \times 6 = 18$$

Jeff bought 3 bags of apples with 5 in each. Jeff's mom bought 1 more bag with 5 in it.



Changing 3×5 to 3×6 : Four representations

Jeffy, Jeff's twin brother,
bought 3 bags of apples
and 6 apples in each.

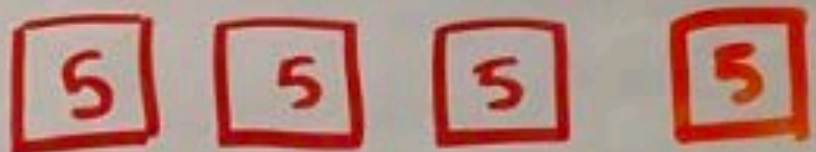


Michelle Sirois' 4th grade

Part 2: Maeve changes the box representation

Part 3: Abdoulaye changes his story problem

Jeff bought 3 bags
of apples with 5 in
each. Jeff's mom bought
1 more bag with 5 in
it.



Jeffy Jeff's twin
brother bought ³ apples
~~bags of apples and~~
~~6 apples in each.~~
and put them in
~~Jeff's~~ three bags
that
have
apples.



Individual and collective agency

- Where do you see individual mathematical agency?
- Where do you see collective mathematical agency?
- How do these examples of agency move the mathematics forward?
- What do you see the teacher doing that is supporting individual and collective agency on the part of the students?

Natasha Gordon's 1st grade

Ms. Schooler has 12 bottle caps. Ms. Gordon has 8 bottle caps.
When they put their collections together, they have 20.

Ms. Schooler found another bottle cap. Ms. Gordon still has 8 bottle caps but Ms. Schooler now has 13 bottle caps. How many do they have together?

Livia's work

Natasha Gordon's 1st grade

Aspects of collective agency

- What characterizes collective mathematical agency?
- Are there different types of collective agency?
- Is it the same across the grade range or are there differences for younger or older students?

The role of leaders

- How can teacher leaders, coaches, and/or administrators support teachers to teach in ways that allow both individual and collective agency?
- Is this important to your work? If so, how?

What we have been noticing

- Collective agency depends on and supports students taking on their own individual agency.
- Students offer their thoughts, both well formulated and not yet fully formed.
- Students share questions and confusions.
- Students work to understand classmates' responses and take responsibility for helping classmates understand.
- Students imply collective work in their use of pronouns: “our conjecture,” “what we’ve figured out,” “you guys are helping me.”
- The class takes ownership of individuals' statements and representations. For example, they revise a classmate's diagram.

Susan Jo
Russell

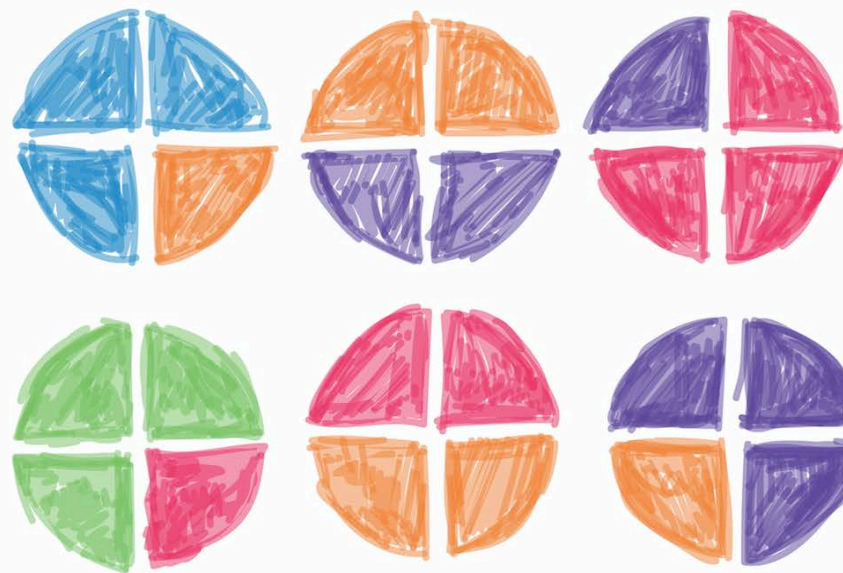
Deborah
Schifter

Reva
Kasman

Virginia
Bastable

Traci
Higgins

But *Why* Does It Work?



Mathematical Argument in the Elementary Classroom

Heinemann

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